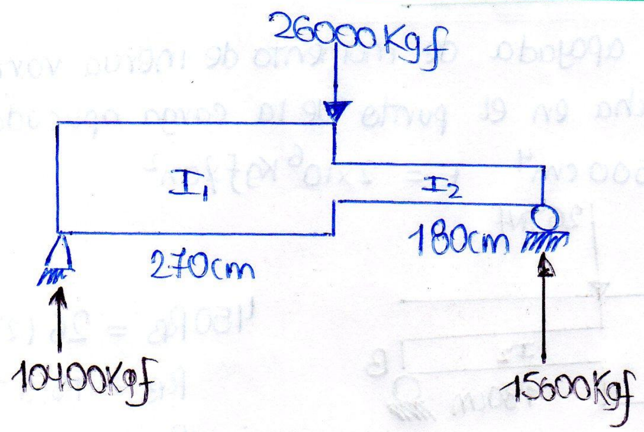
SINGULARIDAD



$$I_1Ey_1'' = 10400x$$
$$I_1Ey_1' = 10400x^2/2 + C_1$$
$$I_1Ey_1 = 10400x^3/6 + C_1x + C_2$$

$$I_2Ey_2'' = 10400x_2 - 26000\langle x_2 - 270\rangle$$
$$I_2Ey_2' = 10400x_2^2/2 - 26000\langle x_2 - 270\rangle^2/2 + C_3$$
$$I_2Ey_2 = 10400x^3/6 - 26000\langle x_2 - 270\rangle^3/6 + C_3x_2 + C_4$$

√ Para $x_1 = 270cm$ $x_2 = 270cm$ $y_1 = y_2$

$$10400\frac{(270)^3}{6} + 270C_1 + \cancel{C_2}^{0} = \left(10400\frac{(270)^3}{6} + 270C_3 + C_4\right)\left(\frac{16680}{12500}\right)$$

$$\boxed{270C_1 - 359.64C_3 - 1.332C_4 - 1.13269104\times10^{10} = 0}$$

√ Para $x_1 = 270cm$ $x_2 = 270cm$ $y_1' = y_2'$

$$10400\frac{(270)^2}{2} + C_1 = \frac{16650}{12500}\left(10400\frac{(270)^2}{2} + C_3\right)$$

$$\boxed{C_1 - 1.332C_3 - 125854560 = 0}$$

√ Para $x_2 = 450cm$ $y_2 = 0$

$$0 = 10400\frac{(450)^3}{6} - 26000\frac{(180)^3}{6} + 450C_3 + C_4$$

$$\boxed{0 = 450C_3 + C_4 + 1.32678\times10^{11}}$$

$C_1 = -317214144$
$C_3 = -332634162.2$
$C_4 = 1.700737297\times10^{10}$

deflexión $x = 270cm$

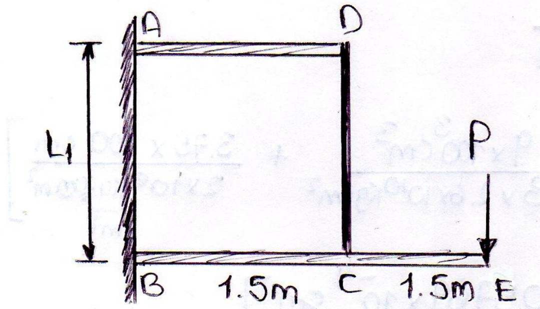
$y_1 = -1.547466\ cm$

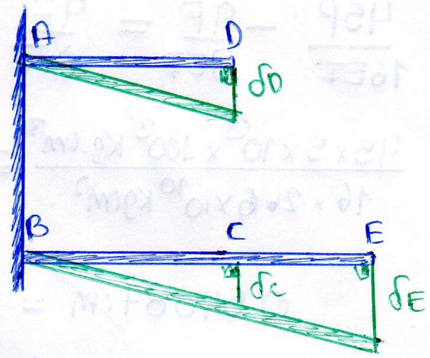
giro $x = 270cm$

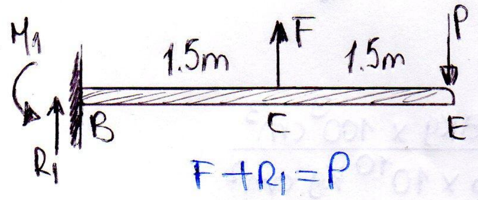
$\theta = 1.857834\times10^{-3}$

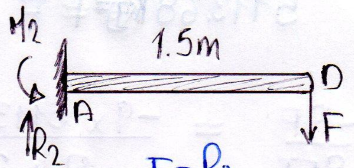
Singularidad - Superposición

Dos vigas en cantilever AD y BE de igual rigidez a la flexión $EI = 2.6 \times 10^{10}$ Kg×cm² están conectadas por una varilla de acero DC para la cual $E_1 = 2 \times 10^6$ Kg cm², $A_1 = 3$ cm² y $L_1 = 3.75$ m. Se pide hallar la flecha del voladizo en "D" debido a la fuerza $P = 5$ TN.









$$F + R_1 = P$$
$$3P = 1.5P + M_1$$

$$F = R_2$$
$$M_2 = 1.5F$$

$$IEy_1'' = R_1 x - M_1 + F\langle x - 1.5\rangle$$
$$IEy_1' = \frac{R_1 x^2}{2} - M_1 x + \frac{F}{2}\langle x-1.5\rangle + C_1$$
$$IEy_1 = \frac{R_1 x^3}{6} - \frac{M_1 x^2}{2} + \frac{F}{2}\langle x-1.5\rangle + C_1 x + C_2$$

$$IEy_2'' = R_2 x - M_2$$
$$IEy_2' = \frac{R_2 x^2}{2} - M_2 x + C_3$$
$$IEy_2 = \frac{R_2 x^3}{6} - \frac{M_2 x^2}{2} + C_3 x + C_4$$

Para $x = 0$ $y_1 = 0$ $C_2 = 0$
$x = 0$ $y_1' = 0$ $C_1 = 0$

Para $x = 0$ $y_2 = 0$ $C_4 = 0$
Para $x = 0$ $y_2' = 0$ $C_3 = 0$

Para $x = 1.5$ $y_1 = \delta_C$

$$y_1 = \frac{9R_1}{16EI} - \frac{9M_1}{8EI}$$
$$y_1 = \frac{9P}{16EI} - \frac{9F}{16EI} - \frac{27P}{8EI} + \frac{27F}{16EI}$$
$$\boxed{\delta_C = \frac{9F}{8EI} - \frac{45P}{16EI}}$$

Para $x = 1.5$ m $y_2 = \delta_D$

$$y_2 = \frac{9R_2}{16EI} - \frac{9M_2}{8EI}$$
$$y_2 = \frac{9F}{16EI} - \frac{27F}{16EI}$$
$$\boxed{\delta_D = \frac{-9F}{8EI}}$$

$$\delta_C = \frac{9F}{16EI} - \frac{45P}{16EI} \rightsquigarrow \delta_C = \frac{45P}{16EI} - \frac{9F}{8EI} \ (\downarrow)$$

$$\delta_D = -\frac{9F}{8EI} \rightsquigarrow \delta_D = \frac{9F}{8EI} \ (\downarrow)$$

$$\delta_C = \delta_D + \delta_{cable}$$

$$\frac{45P}{16EI} - \frac{9F}{8EI} = \frac{9F}{8EI} + \frac{FL_1}{AE_1}$$

$$\frac{45 \times 5 \times 10^3 \times 100^3 \ Kg.cm^3}{16 \times 2.6 \times 10^{10} \ Kgcm^2} = F\left[\frac{9 \times 100^3 \ cm^3}{8 \times 2.6 \times 10^{10} \ Kgcm^2} + \frac{3.75 \times 100 \ cm}{2 \times 10^6 \ \frac{Kg}{cm^2} \times 3cm^2}\right]$$

$$0.54087 \cancel{cm} = F\left[1.05769 \times 10^{-4} \ \frac{\cancel{cm}}{Kg}\right]$$

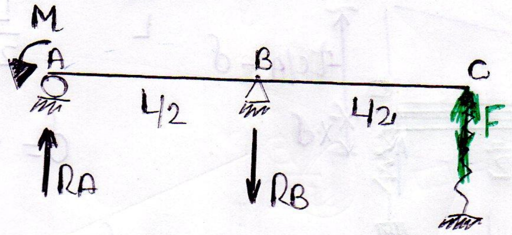
$$5113.68 \ Kg = F$$

$$\delta_D = -\frac{9F}{8EI} = \frac{-9 \times 5113.68Kg \times 100^3 \ cm^3}{8 \times 2.6 \times 10^{10} \ Kg \ cm^2}$$

$$\boxed{\delta_D = 0.22127 \ cm} \ \text{RPTA}$$

Singularidad

La viga ABC tiene una rigidez a la flexión EI y una longitud L. El extremo C esta unido a un resorte de constante K. determinar la fuerza en el resorte debido al momento aplicado en A.



$M_A = 0 \quad 2F + \frac{2M}{L} = R_B$

$M_B = 0 \quad F + \frac{2M}{L} = R_A$

$$IEy'' = R_A x - M - R_B \langle x - \tfrac{L}{2} \rangle$$

$$IEy' = \frac{R_A x^2}{2} - Mx - \frac{R_B}{2}\langle x - \tfrac{L}{2} \rangle^2 + C_1$$

$$IEy' = \frac{R_A x^3}{6} - \frac{Mx^2}{2} - \frac{R_B}{6}\langle x - \tfrac{L}{2}\rangle^3 + C_1 x + C_2$$

Para $x = 0 \quad C_2 = 0$

Para $x = \frac{L}{2} \quad y = 0$

$$C_1 = \frac{ML}{4} - \frac{R_A L^3}{24}$$

Para $x = L$

$$IEy = \frac{R_A L^3}{8} - \frac{R_B L^3}{48} - \frac{ML}{4}$$

$$IEy = \frac{FL^3}{8} + \frac{ML^2}{4} - \frac{FL^3}{24} - \frac{ML^2}{24} - \frac{ML^2}{4}$$

$$IEy = \frac{FL^3}{12} - \frac{ML^2}{24}$$

$$y = \frac{FL^3}{12EI} - \frac{ML^2}{24EI}$$

$$y = -\left[-\frac{FL^3}{12EI} + \frac{ML^2}{24EI}\right]$$

$$y = \frac{ML^2}{24EI} - \frac{FL^3}{12EI}$$

$$y = -\frac{FL^3}{12EI} + \frac{ML^2}{24EI} (\downarrow) \Rightarrow X = \frac{F}{K} \downarrow$$

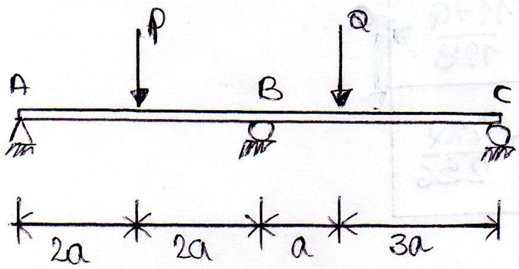
$$\frac{F}{K} + \frac{FL^3}{12EI} = \frac{ML^2}{24EI}$$

$$F\left[\frac{12EI + KL^3}{12KEI}\right] = \frac{ML^2}{24EI}$$

$$F = \frac{KML^2}{24EI + 2KL^3}$$

SINGULARIDAD

Para la viga ABC mostrada, donde EI es constante, determinar la relación entre las fuerzas P y Q, para la fuerza cortante en "c" sea siempre negativa.



$$IEy'' = R_A X - P\langle X-2a\rangle + R_B\langle X-4a\rangle - Q\langle X-5a\rangle$$

$$IEy' = \frac{R_A X^2}{2} - \frac{P}{2}\langle X-2a\rangle^2 + \frac{R_B}{2}\langle X-4a\rangle^2 - \frac{Q}{2}\langle X-5a\rangle^2 + C_1$$

$$IEy = \frac{R_A X^3}{6} - \frac{P}{6}\langle X-2a\rangle^3 + \frac{R_B}{6}\langle X-4a\rangle^3 - \frac{Q}{6}\langle X-5a\rangle^3 + C_1 X + C_2$$

Para $x=0$ $y=0$ $C_2=0$

Para $x=4a$ $y=0$ $C_1=?$

$$0 = \frac{32R_A}{3}a^3 - \frac{4P}{3}a^3 + C_1(4a)$$

$$\boxed{C_1 = \frac{P}{3}a^2 - \frac{8R_A}{3}a^2}$$

Para $x=8a$ $y=0$

$$0 = \frac{256 R_A a^3}{3} - 36Pa^3 + \frac{32R_B a^3}{3} - \frac{9Qa^3}{2} + \frac{8Pa^3}{3} - \frac{64R_A}{3}a^3$$

$$\boxed{0 = 64R_A + \frac{32R_B}{3} - \frac{100P}{3} - \frac{9Q}{2}} \quad \ldots (I)$$

ESTÁTICA

$$8R_A(a) + 4R_B(a) - 6Pa - 3Qa = 0$$

$$8R_A + 4R_B - 6P - 3Q = 0$$

$$\boxed{64R_A + 32R_B - 48P - 24Q = 0} \quad \ldots (II)$$

(II) – (I)

$$\frac{64R_B}{3} - \frac{44P}{3} - \frac{39Q}{2} = 0$$

$$\boxed{R_B = \frac{11P}{16} + \frac{117Q}{128}}$$

$$\boxed{R_A = \frac{13P}{32} - \frac{21Q}{256}}$$

ESTÁTICA

$$R_A + R_B + R_C = P + Q$$

$$\boxed{R_C = \frac{43Q}{256} - \frac{3P}{32}}$$

$$R_C > 0$$

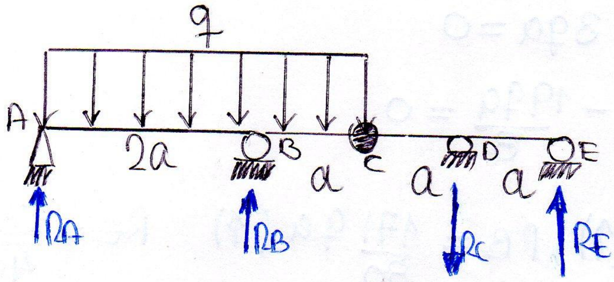
$$\frac{43Q}{256} - \frac{3P}{32} > 0$$

$$\frac{43Q}{256} > \frac{3P}{32}$$

$$\boxed{\frac{43}{24} > \frac{P}{Q}}$$ RPTA

Singularidad

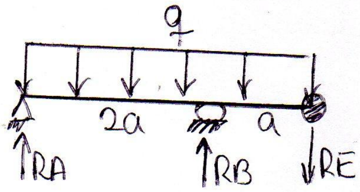
Conociendo a, E, q, e I ; calcular la deflexión bajo la rótula

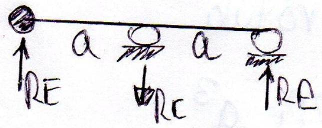


$$\boxed{3R_A + R_B = 4.5qa}$$

$$2R_E = R_C$$

$$\boxed{R_A + R_B - R_E = 3qa}$$





$$IEY_1'' = R_A x - \frac{qx^2}{2} + R_B\langle x-2a\rangle$$

$$IEY_1' = \frac{R_A x^2}{2} - \frac{qx^3}{6} + \frac{R_B}{2}\langle x-2a\rangle^2 + C_1$$

$$IEY_1 = \frac{R_A x^3}{6} - \frac{qx^4}{24} + \frac{R_B}{6}\langle x-2a\rangle^3 + C_1x + C_2$$

Para $x=0$ $y=0$ $C_2=0$

Para $x=2a$ $y=0$ $C_1=?$

$$0 = \frac{8R_A}{6}a^3 - \frac{16qa^4}{24} + 2aC_1$$

$$\boxed{C_1 = \frac{16}{48}qa^3 - \frac{8R_A a^2}{12}}$$

Para $x=3a$ $y_1=y_2$

$$\frac{27R_A}{6}a^3 - \frac{81qa^4}{24} + \frac{R_B a^3}{6} + qa^4 - 2R_A a^3$$

$$EIy_1 = \frac{5R_A}{2}a^3 + \frac{R_B a^3}{6} - \frac{19}{8}qa^4$$

$$\boxed{\frac{5}{2}R_A + \frac{R_B}{6} - \frac{2R_E}{3} - \frac{19}{8}qa = 0}$$

$$IEY_2'' = R_E x - R_C\langle x-a\rangle$$

$$IEY_2' = \frac{R_E x^2}{2} - \frac{R_C}{2}\langle x-a\rangle^2 + C_3$$

$$IEY_2 = \frac{R_E x^3}{6} - \frac{R_C}{6}\langle x-a\rangle^3 + C_3x + C_4$$

Para $x=a$ $y=0$

$$0 = \frac{R_E a^3}{6} + C_3 a + C_4$$

Para $x=2a$ $y=0$

$$0 = \frac{8R_E}{6}a^3 - \frac{R_C}{6}a^3 + 2aC_3 + C_4$$

$$\boxed{C_3 = -\frac{5R_E a^2}{6}}$$

$$\boxed{C_4 = \frac{2R_E a^3}{3}}$$

Para $x=0$ $y_2=y_1$

$$EIy_2 = C_4 = \frac{2R_E a^3}{3}$$

$$3R_A + R_B - 4.5qa = 0$$

$$R_A + R_B - R_E - 3qa = 0$$

$$\frac{5R_A}{2} + \frac{R_B}{6} - \frac{2R_E}{3} - \frac{19qa}{8} = 0$$

$$R_A = \frac{63}{80}qa\ (\uparrow) \quad R_B = \frac{171}{80}qa\ (\uparrow) \quad R_E = \frac{3qa}{40}\ (\downarrow)$$

deflexión rótula

$$\delta_C = \frac{2R_E}{3EI}a^3$$

$$\delta_C = +\frac{2}{3EI}\left(-\frac{3qa}{40}\right)a^3$$

$$\delta_C = -\frac{qa^4}{20EI}$$

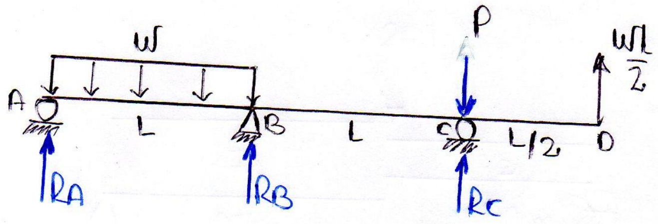
$$\boxed{\delta_C = \frac{qa^4}{20EI}\ (\downarrow)}$$

RPTA.

Comprobado por Área de momento

Singularidad

Utilizando el método funciones de singularidad, calcular los momentos en los apoyos y las reacciones. dibujar los diagramas de DFC y DMF.



$$IEy'' = R_A x - \frac{wx^2}{2} + R_B\langle x-L\rangle + \frac{w\langle x-L\rangle^2}{2} - P\langle x-2L\rangle + R_C\langle x-2L\rangle$$

$$IEy' = \frac{R_A x^2}{2} - \frac{wx^3}{6} + \frac{R_B}{2}\langle x-L\rangle^2 + \frac{w\langle x-L\rangle^3}{6} - \frac{P}{2}\langle x-2L\rangle^2 + \frac{R_C}{2}\langle x-2L\rangle^2 + C_1$$

$$IEy = \frac{R_A x^3}{6} - \frac{wx^4}{24} + \frac{R_B}{6}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 - \frac{P}{6}\langle x-2L\rangle^3 + \frac{R_C}{6}\langle x-2L\rangle^3 + C_1 x + C_2$$

Para $x=0$ $y=0$ $C_2=0$

Para $x=L$ $y=0$ $C_1 = \frac{wL^3}{24} - \frac{R_A L^2}{6}$

Para $x=2L$ $y=0$ $0 = 24R_A + 4R_B - 13wL$

$M_C=0$ $0 = 2R_A + R_B - \frac{7}{4}wL$

$\therefore$

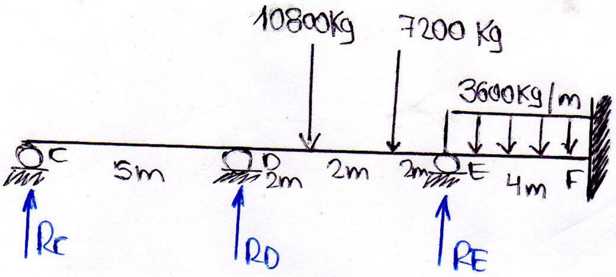
$R_A = \frac{3wL}{8}$

$R_B = wL$

$R_C = P - \frac{7wL}{8}$

Singularidad

Para la viga cargada como se indica, determine los DFC y DMF. Aplique el método de funciones de singularidad. EI = cte



$$IEY'' = R_C X + R_D\langle X-5\rangle - 10800\langle X-7\rangle - 7200\langle X-9\rangle + R_E\langle X-11\rangle - 3600\frac{\langle X-13\rangle^2}{2}$$

$$IEY' = \frac{R_C X^2}{2} + \frac{R_D}{2}\langle X-5\rangle^2 - 5400\langle X-7\rangle^2 - 3600\langle X-9\rangle^2 + \frac{R_E}{2}\langle X-11\rangle^2 - 600\langle X-13\rangle^3 + C_1$$

$$IEY = \frac{R_C X^3}{6} + \frac{R_D}{6}\langle X-5\rangle^3 - 1800\langle X-7\rangle^3 - 1200\langle X-9\rangle^3 + \frac{R_E}{6}\langle X-11\rangle^3 - 150\langle X-13\rangle^4 + C_1 X + C_2$$

Para $X=0$ $y=0$ $C_2=0$

Para $X=5$ $y=0$ $C_1 = -\frac{25}{6}R_C$

Para $X=11$ $y=0$ $0 = 44R_C + 9R_D - 31200$

Para $X=15$ $y'=0$ $0 = 325R_C + 150R_D + 24R_E - 1940000$

Para $X=15$ $y=0$ $0 = 375R_C + 125R_D + 8R_E - 887400$

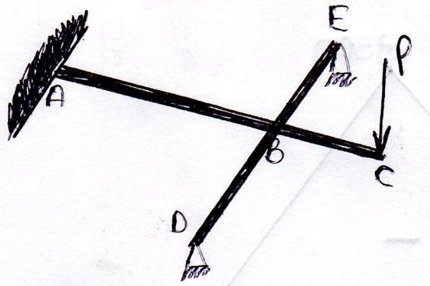
$R_C = -1474$ Kg

$R_D = +10672.89$

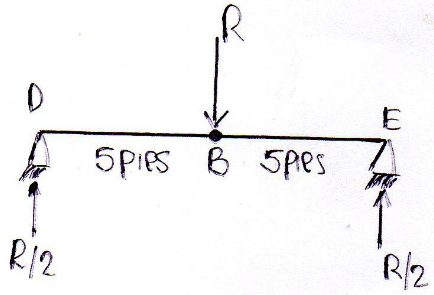
$R_E = +13254.86$

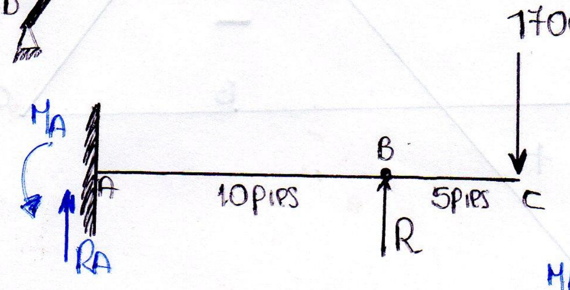
Singularidad

La viga ABC esta empotrada en "A" y se apoya en el punto medio de la viga DE. Lo distancia de A a B es de 10 pies, la distancia de B a C es 5 pies y la longitud de la viga DE es 10 pies, ambas vigas tienen la misma rigidez (EI). dibujar los diagramas de fuerza cortante y momento flector de la viga ABC, sabiendo que P = 1700 Libras.



AC Sobre DB





$R_A = 1700 - R$

$M_A = 25500 - 10R$

$IEY_1'' = \frac{R}{2}X - R\langle X-5\rangle$

$IEY_1' = \frac{R}{2}\cdot\frac{X^2}{2} - R\frac{\langle X-5\rangle^2}{2} + C_1$

$IEY_1 = \frac{RX^3}{12} - \frac{R\langle X-5\rangle^3}{6} + C_1X + C_2$

Para $x=0$ $y_1=0$ $C_2=0$

Para $x=10$ $y_1=0$

$0 = \frac{R(10)^3}{12} - \frac{R(5)^3}{6} + 10C_1$

$C_1 = -\frac{25}{4}R$

deflexion B $\rightarrow$ Para $x=5$

$EIy_B = \frac{R(5)^3}{12} - \frac{25R}{4}(5)$

$y_B = -\frac{125R}{6EI}$

$IEY_2'' = (1700-R)x - M_A + R\langle x-10\rangle$

$IEY_2' = (1700-R)\frac{x^2}{2} - M_Ax + R\frac{\langle x-10\rangle^2}{2} + C_3$

$IEY_2 = (1700-R)\frac{x^3}{6} - \frac{M_Ax^2}{2} + \frac{R\langle x-10\rangle^3}{6} + C_3x + C_4$

Para $x=0$ $y_2=0$ $y_2'=0$ $C_3=0$ $C_4=0$

deflexion B Para $x=10$

$IEy_B = (1700-R)\frac{(10)^3}{6} - \frac{M_A(10^2)}{2}$

$IEy_B = \frac{850000}{6} - \frac{500R}{3} - 1275000 + 500R$

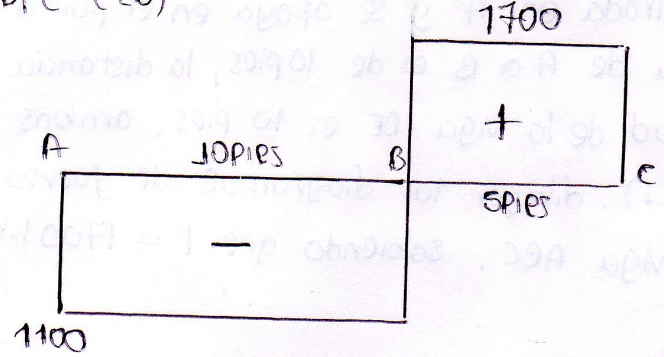
$y_B = \frac{1000R}{3EI} - \frac{2975000}{3EI}$

$\therefore \frac{1000R}{3\cancel{EI}} - \frac{2975000}{3\cancel{EI}} = -\frac{125R}{6\cancel{EI}}$

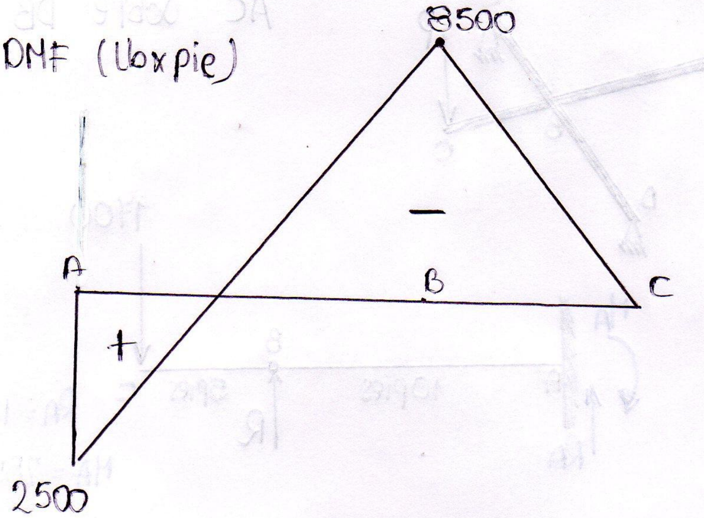
$\frac{2125R}{6} = \frac{2975000}{3}$

$R = 2800\ lb$

DFC (lb)

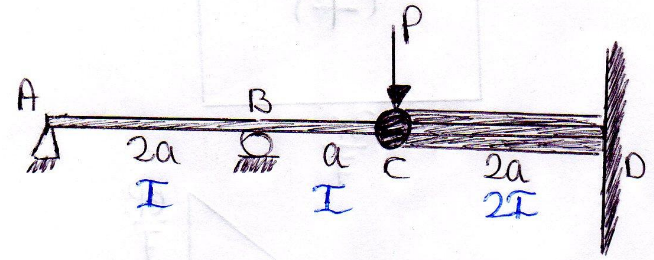


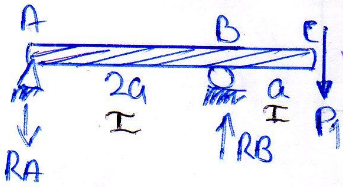
DMF (lbxpie)



Singularidad

La viga ABCD mostrada, tiene una rótula en C y esta empotrada en D. Utilizando el método de funciones de singularidad, dibujar los diagramas de fuerza cortante y momento flector.

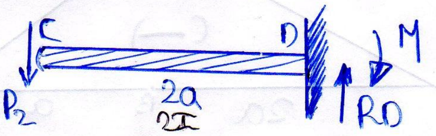




$\checkmark\ R_A + P_1 = R_B$

$\checkmark\ P_1 = 2R_A$

$\therefore\ 3R_A = R_B$



$$IEy_1'' = -R_A x + R_B\langle x-2a\rangle$$

$$IEy_1' = -\frac{R_A x^2}{2} + \frac{R_B}{2}\langle x-2a\rangle^2 + C_1$$

$$IEy_1 = -\frac{R_A x^3}{6} + \frac{R_B}{6}\langle x-2a\rangle^3 + C_1 x + C_2$$

Para $x=0$ $\quad y_1 = 0 \quad C_2 = 0$

Para $x = 2a$ $\quad y_1 = 0$

$$0 = -\frac{R_A(2a)^3}{6} + C_1(2a)$$

$$\boxed{C_1 = \frac{2R_A}{3}a^2}$$

$$2EIy_2'' = -P_2 x$$

$$2EIy_2' = -\frac{P_2 x^2}{2} + C_3$$

$$2EIy_2 = -\frac{P_2 x^3}{6} + C_3 x + C_4$$

Para $x = 2a$ $\quad y_2' = 0$

Para $x = 2a$ $\quad y_2 = 0$

$$\boxed{C_3 = +2P_2 a^2}$$

$$\boxed{C_4 = -\frac{8P_2}{3}a^3}$$

Para $x_1 = 3a \quad x_2 = 0 \quad y_1 = y_2$

$$-\frac{27R_A}{6}a^3 + \frac{R_B}{6}a^3 + 2R_A a^3 = -\frac{8P_2}{6}a^3$$

$$-\frac{5}{2}R_A + \frac{R_B}{6} + \frac{8P_2}{6} = 0$$

$$-2R_A = -\frac{8}{6}P_2 \rightarrow \boxed{R_A = \frac{2P_2}{3}}$$

$\therefore\ P_1 + P_2 = P$

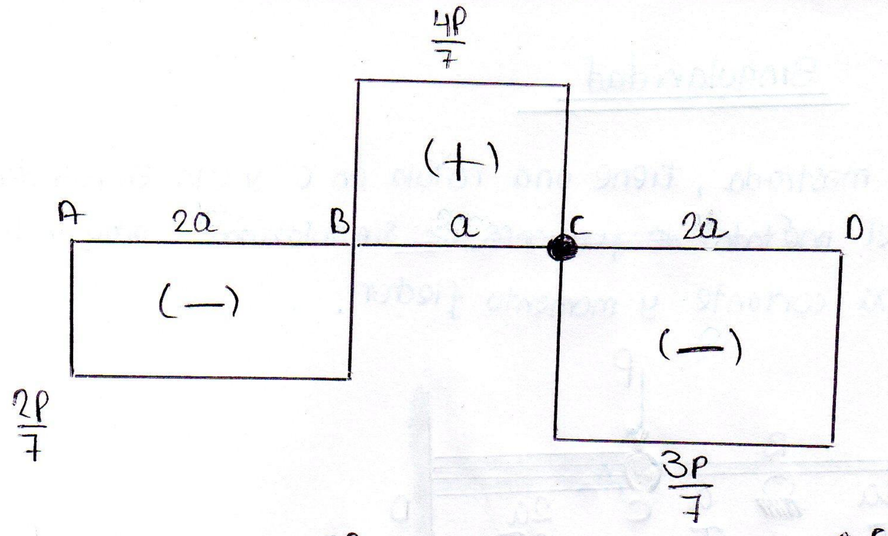
$$2R_A + \frac{3R_A}{2} = P$$

$$\frac{7R_A}{2} = P$$

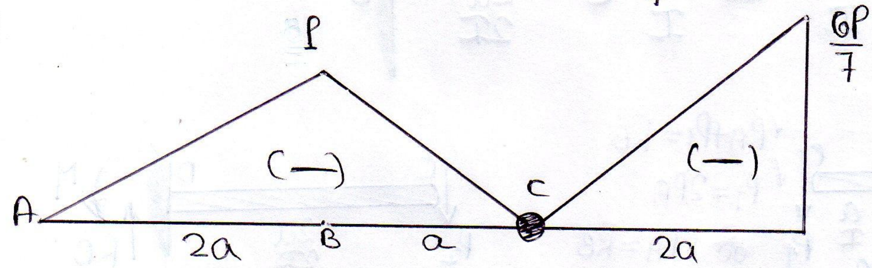
$\checkmark\ P_2 = R_D$

$\checkmark\ M = P_2(2a)$

$R_A = \frac{2P}{7}(\downarrow)$	$R_B = \frac{6P}{7}(\uparrow)$	$R_D = \frac{3P}{7}(\uparrow)$	$M = \frac{6P}{7}$



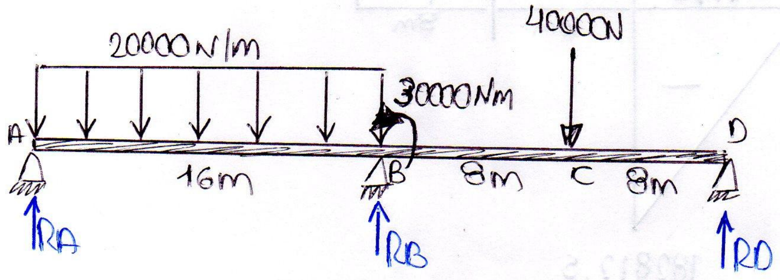
4P/7
(+)
A
2a
B
a
C
2a
D
(—)
(—)
2P/7
3P/7



P
6P/7
(—)
(—)
A
C
2a
B
a
2a

SINGULARIDAD

La viga mostrada Tiene $EI = cte$. Dibujar el diagrama de fuerza cortante y momento flector



$$IEy'' = R_D x - 40000\langle x-8\rangle + 30000\langle x-16\rangle^0 + R_B\langle x-16\rangle - 20000\frac{\langle x-16\rangle^2}{2}$$

$$IEy' = \frac{R_D x^2}{2} - 20000\langle x-8\rangle^2 + 30000\langle x-16\rangle + \frac{R_B}{2}\langle x-16\rangle^2 - \frac{10000}{3}\langle x-16\rangle^3 + C_1$$

$$IEy = \frac{R_D x^3}{6} - \frac{20000}{3}\langle x-8\rangle^3 + 15000\langle x-16\rangle^2 + \frac{R_B}{6}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-16\rangle^4 + C_1 x + \cancel{C_2}^{\,0}$$

Para $x = 16m$ $\quad y = 0$

$$0 = \frac{2048 R_D}{3} - \frac{10240000}{3} + 16C_1$$

$$\boxed{C_1 = \frac{640000}{3} - \frac{128 R_D}{3}}$$

Para $x = 32$ $\quad y = 0$

$$0 = \frac{16384 R_D}{3} - 92160000 + 3840000 + \frac{2048 R_B}{3} - \frac{163840000}{3} + 32C_1$$

$$0 = \frac{16384 R_D}{3} + \frac{2048 R_B}{3} - 142933333.3 + 6826666.667 - \frac{4096 R_D}{3}$$

$$\boxed{0 = 4096 R_D + \frac{2048 R_B}{3} - 136106666.7}$$

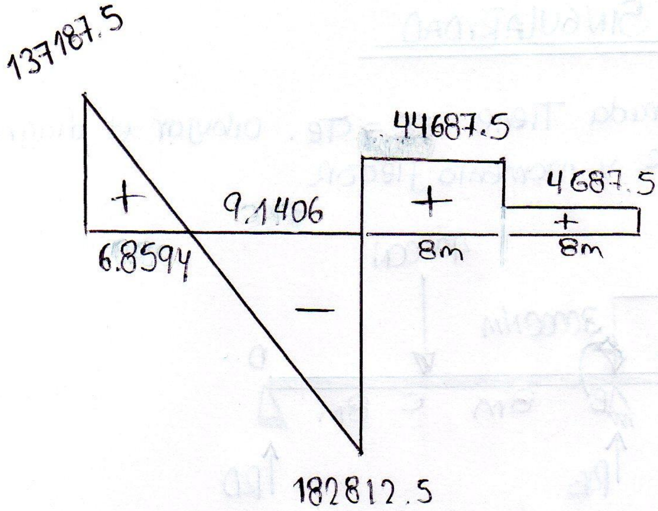
ESTÁTICA

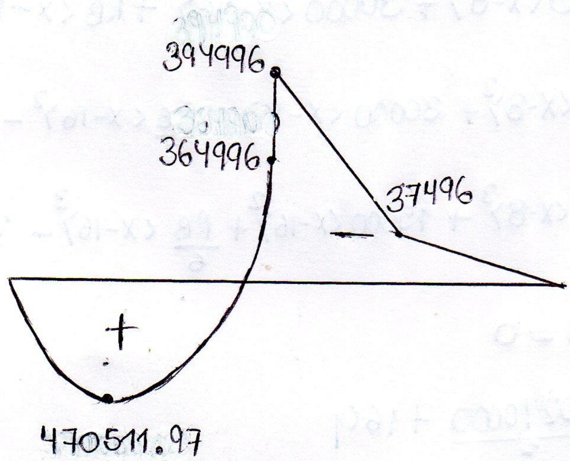
$$\boxed{32 R_D + 16 R_B - 3490000 = 0}$$

$R_D = -4687.5$

$R_B = +227500$

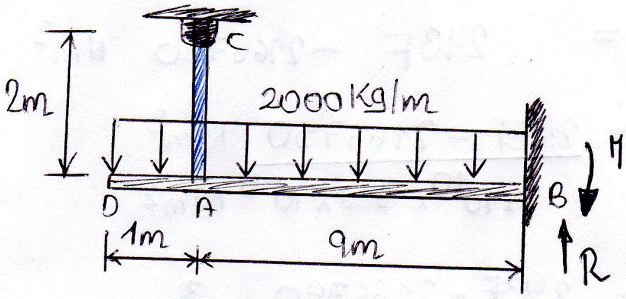
$R_A = +137187.5$


137187.5
44687.5
4687.5
+
9.1406
+
+
6.8594
8m
8m
−
182812.5


394996
364996
37496
−
+
470511.97

Singularidad

Para el sistema mostrado, formado por una viga (DAB) y un cable (CA) ambos de acero ($E = 2\times10^6\ Kg/cm^2$). se solicita dibujar los diagramas de fuerza cortante y momento flector de la viga DAB, sabiendo que para la viga $I = 500cm^4$ y para el cable $A = 5cm^2$



$$IEy'' = -\frac{2000x^2}{2} + F\langle x-1\rangle$$

$$IEy' = -\frac{2000x^3}{6} + \frac{F}{2}\langle x-1\rangle^2 + C_1$$

$$IEy = -\frac{2000x^4}{24} + \frac{F}{6}\langle x-1\rangle^3 + C_1x + C_2$$

Para $x=10$ $y'=0$

$$C_1 = \frac{2000(10^3)}{6} - \frac{F(9)^2}{2}$$

$$\boxed{C_1 = \frac{1000000}{3} - \frac{81F}{2}}$$

Para $x=10$ $y=0$

$$0\quad C_2 = \frac{2000(10)^4}{24} - \frac{F(9)^3}{6} - \frac{1000000\times10}{3} + \frac{810F}{2}$$

$$\boxed{C_2 = \frac{567F}{2} - 2500000}$$

√ deflexion en la viga

Para $x_1 = \quad y = ?$

$$IEy = -\frac{2000}{24} + \frac{10^6}{3} - \frac{81F}{2} + \frac{567F}{2} - 2500000$$

$$IEy = 243F - 2166750 \quad m^3$$

$$y = \frac{243F - 2166750}{2\times10^{10} \times 500\times10^{-8}} \; \frac{Kgm^3}{Kgm^2}$$

$$y = \frac{243F - 2166750}{100000} \; m$$

√ deformacion en la barra

$$\delta = \frac{F\times 2}{5\times10^{-4}\times 2\times10^{10}} = 2F\times10^{-7}$$

defor = defle

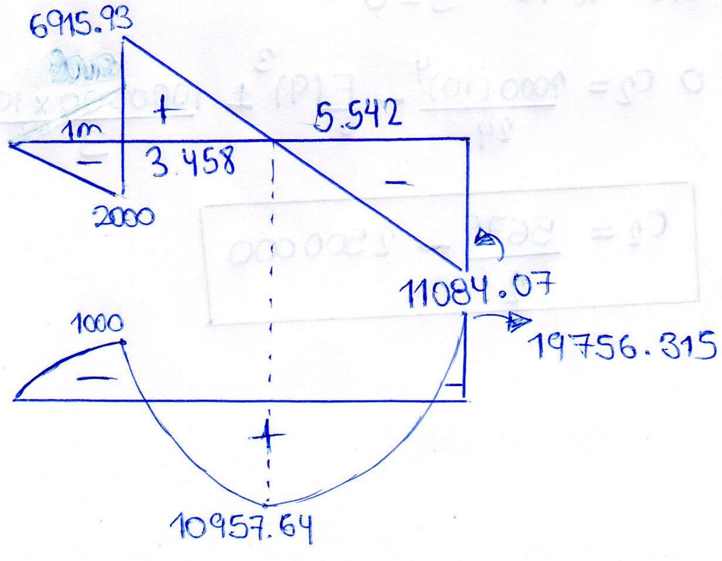
$$2F\times10^{-7} = \frac{243F - 2166750}{10^5}$$

$$0.02F = 243F - 2166750$$

$$2166750 = 243.02F$$

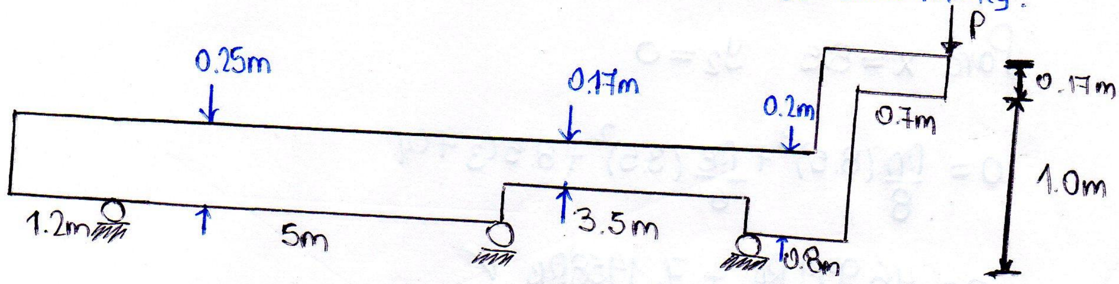
$$F = 8915.93\,Kg \qquad M = 19756.63$$

$$R_A = 11034.07\,Kg$$



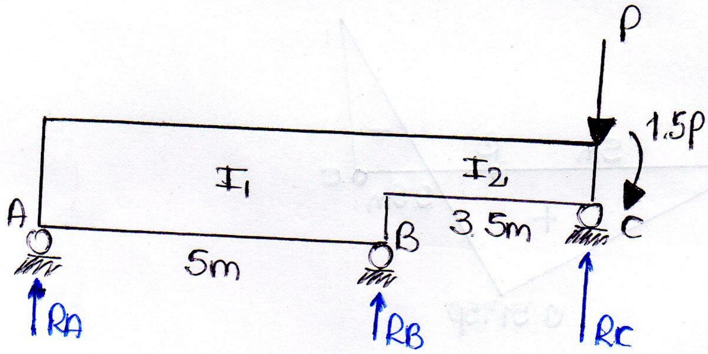
SINGULARIDAD

En la viga mostrada, se pide determinar el valor que debe tener la fuerza Puntual "P", aplicada en el extremo del volado, de manera que en el apoyo "B" se genere un momento flector positivo de 3000 m-Kg.



$$I_1 = \frac{1m \times 0.25^3 m}{12} = 1.302 \times 10^{-3} m^4$$

$$I_2 = \frac{1m \times 0.17^3 m}{12} = 4.094 \times 10^{-4} m^4$$



$$I_1 E y_1'' = R_A x$$

$$I_1 E y_1' = R_A x^2/2 + C_1$$

$$I_1 E y_1 = R_A x^3/6 + C_1 x + C_2$$

0

Para $x = 5$ $y_1 = 0$

$$0 = \frac{R_A x^3}{6} + C_1 x$$

$$\boxed{\frac{-25 R_A}{6} = C_1}$$

$$I_2 E y_2'' = R_A x + R_B \langle x-5 \rangle$$

$$I_2 E y_2' = R_A x^2/2 + R_B \langle x-5 \rangle^2/2 + C_3$$

$$I_2 E y_2 = R_A x^3/6 + R_B \langle x-5 \rangle^3/6 + C_3 x + C_4$$

Para $x = 5$ $y_2 = 0$

$$0 = \frac{R_A x^3}{6} + C_3 x + C_4$$

$$\boxed{C_4 = 28.562 R_A}$$

Para $x = 5$ $y_1' = y_2'$

$$\frac{R_A x^2}{2} + C_1 = \frac{I_1}{I_2}\left(\frac{R_A x^2}{2} + C_3\right)$$

$$\frac{25 R_A}{2} - \frac{25 R_A}{6} = 3.18\left(\frac{25 R_A}{2}\right) + 3.18 C_3$$

$$\boxed{C_3 = -9.879 R_a}$$

- De la estática

$$R_A + R_B + R_C - P = 0$$ ✓

$$0 R_A + 5 R_B + 8.5 R_C - 10P = 0$$ ✓

Para $x = 8.5 \quad y_2 = 0$

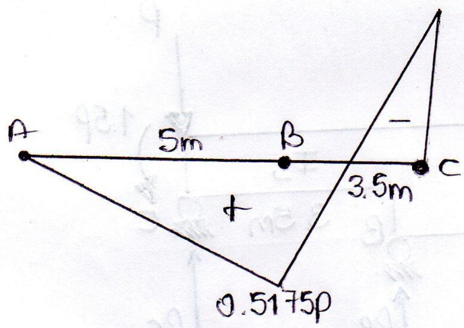
$$0 = \frac{R_A}{8}(8.5)^3 + \frac{R_B}{6}(3.5)^2 + 8.5 C_3 + C_4$$

$$0 = 46.944 R_A + 7.1458 R_B$$ ✓

$R_A = 0.1035P$

$R_B = -0.6799P$

$R_C = 1.5764P$

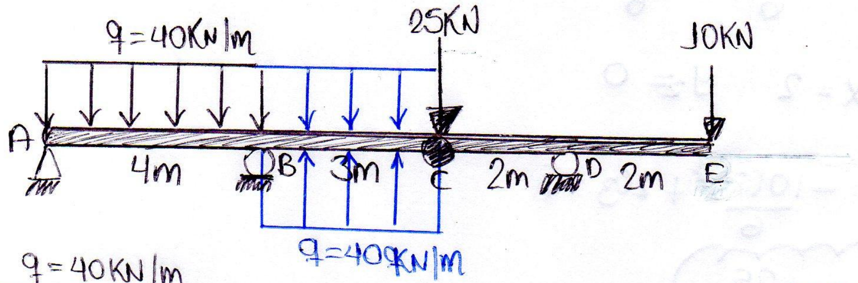


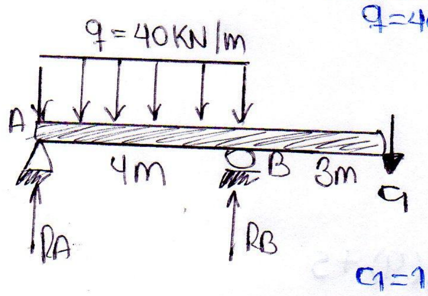
$$0.5175P = 3000 \text{ m-Kg}$$

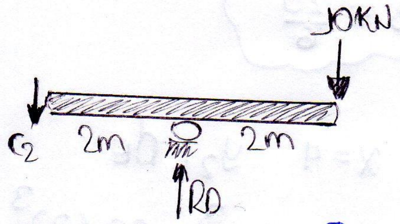
$$P = 5797.10 \text{ Kg}$$

Singularidad

DETERMINAR la deflexión en el punto E de la viga mostrada







$C_1 = 15KN$

$C_2 = 10KN$

$R_D = 20KN$

$7R_A + 3R_B = 800$

$R_A + R_B = 175$

$R_A = 68.75 KN$

$R_B = 106.25 KN$

$IEY_2'' = -10(x) + 20\langle x-2\rangle$

$IEY_2' = -\frac{10x^2}{2} + \frac{20}{2}\langle x-2\rangle^2 + C_3$

$IEY_2 = -\frac{10x^3}{6} + \frac{20}{6}\langle x-2\rangle^3 + C_3x + C_4$

$IEY'' = 68.75x - \frac{40x^2}{2} + 106.25\langle x-4\rangle + \frac{40}{2}\langle x-4\rangle^2$

$IEY_1' = \frac{68.75x^2}{2} - \frac{40x^3}{6} + \frac{106.25}{2}\langle x-4\rangle^2 + \frac{40}{6}\langle x-4\rangle^3 + C_1$

$IEY_1 = \frac{68.75x^3}{6} - \frac{40x^4}{24} + \frac{106.25}{6}\langle x-4\rangle^3 + \frac{40}{24}\langle x-4\rangle^4 + C_1x + C_2$

Para $x_1 = 0 \quad y_1 = 0 \quad C_2 = 0$

Para $x_1 = 4 \quad y_1 = 0$

$$0 = \frac{68.75}{6}(4)^3 - \frac{40(4)^4}{24} + 4C_1$$

$$C_1 = \frac{-230}{3}$$

Para $x = 7 \quad y_1 = ?$

$$EIy = \frac{68.75}{6}(7)^3 - \frac{40(7^4)}{24} + \frac{106.25}{6}(3)^3 + \frac{40}{24}(3^4) + 7C_1$$

$$y = \frac{5}{EI}$$

Para $x_2 = 0 \quad y_2 = ?$

$IEY_2 = C_4$

$$y_2 = \frac{C_4}{EI}$$

$\therefore \; y_2 = y_1$

$$\frac{C_4}{EI} = \frac{5}{EI}$$

$C_4 = 5$

3

$$IEy_2 = -\frac{10x^3}{6} + \frac{20}{6}\langle x-2\rangle^3 + C_3x + 5$$

Para $x = 2 \quad y_2 = 0$

$$0 = -\frac{10(2)^3}{6} + 2C_3 + 5$$

$$C_3 = \frac{25}{6}$$

Para $x = 4 \quad y_2 = \delta_E$

$$IEy_2 = \frac{-10(4)^3}{6} + \frac{20}{6}(2)^3 + \frac{25}{6}(4) + 5$$

$$IEy_2 = -\frac{175}{3}$$

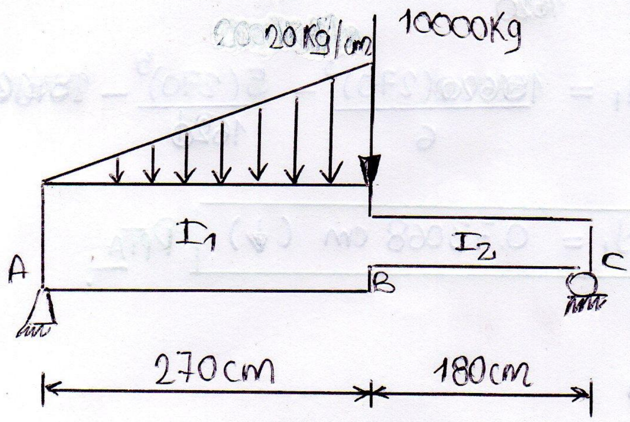
$$\delta_E = \frac{175}{3EI} \; (\downarrow)$$

Rpta.

FUNCION DE SINGULARIDAD

Dada la viga simplemente apoyada de momento de inercia variable, determine el giro y la flecha en el punto de la carga aplicada de 10TNf por el método de funciones de singularidad.

Se conoce $I_1 = 16650\,cm^4$ $I_2 = 12500\,cm^4$ $E = 2\times10^6\,Kg/cm^2$



ESTÁTICA

$$450R_C = 10000(270) + 20\frac{(270)}{2}\frac{(270)}{3}\times 2$$

$$R_C = 7080\ Kg\ (\uparrow)$$

$$R_A = 15620\ Kg\ (\uparrow)$$

$$I_1EY_1'' = R_A x_1 - \frac{20\,x_1}{270}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right)$$

$$I_1EY_1'' = R_A x_1 - \frac{1}{81}x_1^3$$

$$I_1EY_1' = \frac{R_A x_1^2}{2} - \frac{1\,x_1^4}{324} + C_1$$

$$I_1EY_1 = \frac{R_A x_1^3}{6} - \frac{1\,x_1^5}{1620} + C_1x_1 + \cancel{C_2} \rightarrow 0$$

$$I_2EY_2'' = R_C x_2$$

$$I_2EY_2' = \frac{R_C x_2^2}{2} + C_3$$

$$I_2EY_2 = \frac{R_C x_2^3}{6} + C_3x_2 + \cancel{C_4} \rightarrow 0$$

Para $x_1 = 270$ $x_2 = 180$ $y_1 = y_2$

$$\frac{5620(270)^3}{6} - \frac{1(270)^5}{1620} + 270C_1 = \left[\frac{7080(180)^3}{6} + 180C_3\right]\times\frac{16650}{12500}$$

$$\boxed{270C_1 - 239.76C_3 + 8384170680 = 0}$$

Para $x_1 = 270$ $x_2 = 180$ $y_1' = -y_2'$

$$\frac{5620(270)^2}{2} - \frac{1(270)^4}{324} + C_1 = -\left[\frac{7080(180)^2}{2} + C_3\right]\times\frac{16650}{12500}$$

$$\boxed{C_1 + 1.332C_3 + 341221572 = 0}$$

$C_1 = -155120119.2$

$C_3 = -139715805.4$

Hallando deflexión

$$I_1 E Y_1 = \frac{R_A x^3}{6} - \frac{1 x_1^5}{1620} + C_1 x_1$$

$$16650(2\times10^6)\,y_1 = \frac{5620(270)^3}{6} - \frac{5(270)^5}{1620} - 155120119.2(270)$$

$$\boxed{y_1 = 0.73068 \text{ cm } (\downarrow)}$$ RPTA.

Hallando giro

$$I_1 E Y_1' = \frac{R_A x^2}{2} - \frac{x^4}{324} + C_1$$

$$16650 \times 2\times10^6\, y_1' = \frac{5620(270)^2}{2} - \frac{270^4}{324} - 155120119.2$$

$$\boxed{y_1' = 1.00\times10^{-3} \text{ rad antihorario}}$$ RPTA.